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A Note on p-Nilpotence of Finite Groups

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In [1] the following was proved:

THEOREM 2. *Let P be a Sylow 2-subgroup of a finite group G. Suppose that $\Omega_1(P \cap G')$ is contained in $Z(P)$. If P is quaternion-free and $N_G(P)$ is 2-nilpotent, then G is 2-nilpotent.*

Then in [1] they state they "do not know any examples of groups which show that the quaternion-free hypothesis is necessary in Theorem 2." In this note we give an example to show that the above hypothesis is necessary.

EXAMPLE. $G = GL(2,3)$. The elements $a = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 1 \end{smallmatrix}\right)$, $b = \left(\begin{smallmatrix} 2 & 0 \\ 2 & 1 \end{smallmatrix}\right)$, $c = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$ generate $GL(2,3)$, and the following relations hold:

$$a^8 = b^2 = c^3 = 1, \qquad b^{-1}ab = a^3, \qquad c^{-1}a^2c = ab,$$
$$c^{-1}abc = aba^2, \qquad b^{-1}cb = c^2.$$

Thus $P = \langle a, b \rangle$ is a Sylow 2-subgroup of $GL(2,3)$ and is a semidihedral group of order 16. Also $Q = \langle a^2, ab \rangle$ is a quaternion group of order 8 and G is not quaternion-free. Further, the normal subgroups of G are $G, G' = SL(2,3) = Q\langle c \rangle$, $G'' = Q$, $G''' = Z(P) = Z(Q) = \langle a^4 \rangle$, and $\{1\}$. It is easily seen that $N_G(P)$ is 2-nilpotent and $\Omega_1(P \cap G')$ is contained in $Z(P)$. Therefore the quaternion-free hypothesis is necessary in [1, Theorem 2] since G is not 2-nilpotent.

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REFERENCE

1. A. Ballester-Bolinches and Xiuyun Guo, Some results on p-nilpotence and solubility of finite groups, *J. Algebra* **228** (2000), 491–496.